

The role of $a_1(1260)$ in $\pi^- p \rightarrow a_1^-(1260)p$ and $\pi^- p \rightarrow \pi^- \rho^0 p$ reactions near threshold

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We report on a theoretical study of the $\pi^- p \rightarrow a_1^-(1260)p$ and $\pi^- p \rightarrow \pi^- \rho^0 p$ reactions near threshold within an effective Lagrangian approach. The production process is described by t -channel ρ^0 meson exchange. For the $\pi^- p \rightarrow \pi^- \rho^0 p$ reaction, the final $\pi^- \rho^0$ results from the decay of the $a_1(1260)$ resonance which is assumed as a dynamically generated state from the $K^* \bar{K}$ and $\rho\pi$ coupled channel interactions. We calculate the total cross section of the $\pi^- p \rightarrow a_1^-(1260)p$ reaction. It is shown that, with the coupling constant of the $a_1(1260)$ to $\rho\pi$ channel obtained from the chiral unitary theory and a cut off parameter $\Lambda_\rho \sim 1.5$ GeV in the form factors, the experimental measurement can be reproduced. Furthermore, the total and differential cross sections of $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \pi^- \rho^0 p$ reaction are evaluated, and it is expected that our model calculations can be tested by future experiments. These reactions are important for the study of the $a_1(1260)$ resonance and would provide further clue for the nature of $a_1(1260)$ state.

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I. INTRODUCTION

Within the picture of the classical quark model, the mesons are bound states of quarks and antiquarks. This picture is very successful. Most of the known mesons can be described very well within the quark model [1]. However, it seems that the meson spectrum is much richer than that predicted by the quark model. There is a growing set of experimental observations of resonance-like structures with quantum numbers which are forbidden for the quark-antiquark system or situated at masses which cannot be explained by the quark-antiquark model [2, 3]. For example, the new observations [4–10] have challenged the conventional wisdom that mesons are made of quark-antiquark pairs in the low energy region.

In the quark model, the ground state axial-vector resonances are $a_1(1260)$ and $b_1(1235)$ with $I^G(J^{PC}) = 1^-(1^{++})$ and $1^+(1^{+-})$, respectively. The experimental mass $M_{a_1(1260)} = 1230 \pm 40$ MeV is more precisely than its width $\Gamma_{a_1(1260)} = 250 \sim 600$ MeV assigned by the Particle Data Group [1]. A recent COMPASS measurement published in 2010 [11] provides a much smaller uncertainty of the width of $a_1(1260)$ $\Gamma_{a_1(1260)} = 367 \pm 9_{-25}^{+28}$ MeV.

In the chiral unitary approach, the low lying axial-vector mesons, $a_1(1260)$ and $b_1(1235)$, are composite particles of a vector meson and a pseudoscalar meson in coupled channels [12]. Indeed, the $a_1(1260)$ is dynamically generated from the $K^* \bar{K}$ and $\rho\pi$ channels and the couplings of the $a_1(1260)$ to these channels can be also obtained at the same time [12]. Based on these results, the radiative decay of the $a_1(1260)$ axial-vector meson was studied in Refs. [13, 14], where the theoretical calculations agree with the experimental values within uncertainties.

Recently the COMPASS collaboration [9] reported the observation of a resonancelike structure around 1.42 GeV with

axial-vector quantum numbers $1^-(1^{++})$ in the $f_0(980)\pi$ P -wave of the $\pi^-\pi^-\pi^+$ final state, and it was claimed as a signal as a new resonances that was named the “ $a_1(1420)$ ” state with width around 140 MeV. It is very difficult to explain this structure as a new state within the quark model, because the radial excitation of $a_1(1260)$ is expected to have a mass above 1650 MeV. Furthermore, it is not expected that the radial excitation state has a width which is much smaller than the one of the ground state. In Refs. [15, 16], the “ $a_1(1420)$ ” state can be explained as a triangle singularity via the decay of $a_1(1260)$ into $K^* \bar{K}$ and subsequent rescattering of the K from the K^* decay to form the $f_0(980)$ resonance. In Ref. [17], the production of a_1 states are studied in heavy meson decays which can also provide insights to the $a_1(1420)$ and the future experimental analyses will very probably lead to a deeper understanding of the nature of the $a_1(1420)$.

In this work, with the coupling of $a_1(1260)$ to the $\rho\pi$ channel which was obtained within the picture that the $a_1(1260)$ resonance is dynamically generated from the $K^* \bar{K}$ and $\rho\pi$ channels [12], we study the role of $a_1(1260)$ resonance in the $\pi^- p \rightarrow a_1^-(1260)p$ and $\pi^- p \rightarrow \rho^0 \pi^- p$ reactions near threshold using an effective Lagrangian approach. In our calculation, the t channel ρ^0 exchange is considered. The total cross sections of $\pi^- p \rightarrow a_1^-(1260)p$ reaction are calculated. It is found that the theoretical calculations for the total cross sections of $\pi^- p \rightarrow a_1^-(1260)p$ reaction are in agreement with the experimental data. In addition, the total and differential cross sections for the $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \rho^0 \pi^- p$ reaction are predicted and could be tested by future experiments. Because the main decay channel of $a_1(1260)$ resonance is the $\rho\pi$ channel, the $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \rho^0 \pi^- p$ reaction is very useful to deep understanding the nature of $a_1(1260)$ state and also the nature of the $a_1(1420)$.

This paper is organized as follows. In Sec. II, formalism and ingredients used in the calculation are given. In Sec. III, the results are presented and discussed. Finally, a short summary is given in the last section.

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II. FORMALISM AND INGREDIENTS

The combination of effective Lagrangian method and isobar model is an important theoretical approach in describing the meson production processes. In this section, we introduce the theoretical formalism and ingredients to calculate the $a_1(1260)$ hadronic production in $\pi^- p \rightarrow a_1(1260)p$ and $\pi^- p \rightarrow \rho^0 \pi^- p$ reactions within the effective Lagrangian method.

A. Feynman diagrams and interaction Lagrangian densities

The basic tree level Feynman diagrams for the $\pi^- p \rightarrow a_1(1260)p$ and $\pi^- p \rightarrow \rho^0 \pi^- p$ reactions are depicted in Fig. 1 and Fig. 2, respectively. For these reactions, the t -channel ρ^0 exchange is considered in this calculation, since the main decay channel of $a_1(1260)$ is the $\rho\pi$ channel.

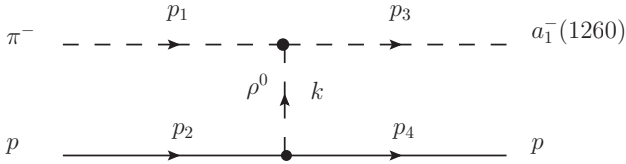


FIG. 1: Feynman diagrams for $\pi^- p \rightarrow a_1(1260)p$ reaction. We also show the definition of the kinematical (p_1 , p_2 , p_3 , p_4 , and k) that we use in the present calculation. In addition, we use $k = p_2 - p_4$.

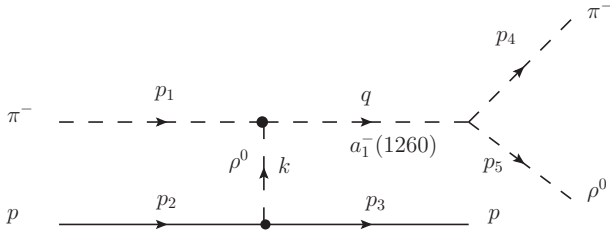


FIG. 2: Feynman diagrams for $\pi^- p \rightarrow \rho^0 \pi^- p$ reaction.

To compute the contributions of diagrams shown in Figs. 1 and 2, we use the interaction Lagrangian density for the ρNN vertex as in Refs. [18–21],

$$\mathcal{L}_{\rho NN} = -g_{\rho NN} \bar{N}(\gamma^\mu - \frac{\kappa_\rho}{2m_N} \sigma^{\mu\nu} \partial_\nu) \vec{\tau} \cdot \vec{\rho}_\mu N, \quad (1)$$

where the parameters are taken as commonly used ones [22–26]: $g_{\rho NN} = 3.36$ and $\kappa_\rho = 6.1$.

In addition, we need also the effective interaction of the $a_1(1260)\rho\pi$ vertex. As mentioned before, in the chiral unitary approach of Ref. [12], the $a_1(1260)$ resonance is dynamically generated from the interaction of $K^* \bar{K}$ and $\rho\pi$ interactions.

One can write down the $\pi^- \rho^0 a_1(1260)$ vertex of Figs. 1 and 2 as,

$$v = \frac{1}{\sqrt{2}} g_{a_1 \rho \pi} \varepsilon^\mu(\rho) \varepsilon_\mu^*(a_1), \quad (2)$$

where $\varepsilon^\mu(\rho)$ and $\varepsilon_\mu^*(a_1)$ are the polarization vectors of ρ and $a_1(1260)$. The $g_{a_1 \rho \pi}$ is the coupling constant of the $a_1(1260)$ to the $\rho\pi$ channel, which is taken to be $(-3795, 2330)$ MeV as obtained in Ref. [12]. The factor $\frac{1}{\sqrt{2}}$ in Eq. (2) accounts for the fact that in the $I = 1$ and $I_3 = -1$ combination of $\rho\pi$ mesons,

$$|\rho\pi\rangle (I = 1, I_3 = -1) = \frac{1}{\sqrt{2}} (\rho^0 \pi^- - \rho^- \pi^0). \quad (3)$$

With the effective Lagrangian densities above, we can straightforwardly construct the invariant scattering amplitude for $\pi^- p \rightarrow a_1(1260)p$ reaction corresponding to the Feynman diagram in Fig. 1:

$$\begin{aligned} \mathcal{M}(\pi^- p \rightarrow a_1(1260)p) &= -\frac{ig_{\rho NN} g_{a_1 \rho \pi} F_\rho(k)}{\sqrt{2}} \bar{u}(p_4, s_4) \\ &\times [\gamma^\mu + \frac{\kappa_\rho}{2m_N} (k^\mu - \not{k} \gamma^\mu)] u(p_2, s_2) G_{\mu\nu}(k) \varepsilon^{*\nu}(p_3, s_3), \end{aligned} \quad (4)$$

where s_4 , s_2 and s_3 are the polarization variables of final proton, initial proton and $a_1(1260)$ resonance, respectively. The ρ -meson propagator $G_{\mu\nu}(k)$ is,

$$G_{\mu\nu}(k) = -i \frac{g_{\mu\nu} - k_\mu k_\nu / m_\rho^2}{k^2 - m_\rho^2}, \quad (5)$$

where m_ρ is the mass of the ρ meson and we take $m_\rho = 775.26$ MeV.

In Eq. (4), $F_\rho(k)$ is the form factor for ρNN vertex and we take it as in Refs. [18, 19],

$$F_\rho(k) = \left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - k^2} \right)^2, \quad (6)$$

with Λ_ρ the cut off parameter which will be discussed in the following.

Similarly, we can get the invariant scattering amplitude for $\pi^- p \rightarrow \pi^- \rho^0 p$ reaction corresponding to the Feynman diagram in Fig. 2:

$$\begin{aligned} \mathcal{M}(\pi^- p \rightarrow \pi^- \rho^0 p) &= -\frac{ig_{\rho NN} g_{a_1 \rho \pi}^2 F_\rho(k) F_{a_1}(q)}{\sqrt{2}} \bar{u}(p_3, s_3) \\ &\times [\gamma^\mu + \frac{\kappa_\rho}{2m_N} (k^\mu - \not{k} \gamma^\mu)] u(p_2, s_2) G_{\mu\nu}(k) G^{\nu\sigma}(q) \varepsilon_\sigma^*(p_5, s_5), \end{aligned} \quad (7)$$

where s_5 is the polarization variable of ρ^0 meson, and $G^{\nu\sigma}(q)$ is the $a_1(1260)$ propagator, which is,

$$G^{\nu\sigma}(q) = -i \frac{g^{\nu\sigma} - q^\nu q^\sigma / m_{a_1}^2}{q^2 - m_{a_1}^2 + im_{a_1} \Gamma_{a_1}}, \quad (8)$$

where Γ_{a_1} and m_{a_1} are the width and mass of the $a_1(1260)$ resonance, respectively. We take $m_{a_1} = 1230$ MeV. For Γ_{a_1} , as mentioned above, since its value has large uncertainties, we take $\Gamma_{a_1} = 250, 425$, and 600 MeV for comparison.

In Eq. (7), $F_{a_1}(q)$ is the form factor of $a_1(1260)$ state. In our present calculation, we adopt the following form as in many previous works [22–26]:

$$F_{a_1}(q) = \frac{\Lambda_{a_1}^4}{\Lambda_{a_1}^4 + (q^2 - m_{a_1}^2)^2}, \quad (9)$$

where Λ_{a_1} is the cutoff parameter of $a_1(1260)$ resonance.

The differential cross section in the center of mass frame (c.m.) for the $\pi^- p \rightarrow a_1^-(1260)p$ and $\pi^- p \rightarrow \rho^0 \pi^- p$ reactions can be derived from the invariant scattering amplitude square $|\mathcal{M}|^2$, reading as:

$$\frac{d\sigma(\pi^- p \rightarrow a_1^-(1260)p)}{d\cos\theta} = \frac{m_p^2}{8\pi W^2} \frac{|\vec{p}_3^{\text{c.m.}}|}{|\vec{p}_1^{\text{c.m.}}|} \times \left(\frac{1}{2} \sum_{s_2} \sum_{s_3, s_4} |\mathcal{M}(\pi^- p \rightarrow a_1^-(1260)p)|^2 \right), \quad (10)$$

where W is the invariant mass of the $\pi^- p$ system, whereas, θ denotes the scattering angle of the outgoing $a_1^-(1260)$ resonance relative to π^- beam direction in the c.m. frame. In the above equation, $\vec{p}_1^{\text{c.m.}}$ and $\vec{p}_3^{\text{c.m.}}$ are the 3-momenta of the initial π^- meson and the final $a_1(1260)$ mesons,

$$|\vec{p}_1^{\text{c.m.}}| = \frac{\lambda^{1/2}(W^2, m_{\pi^-}^2, m_p^2)}{2W}, \quad (11)$$

$$|\vec{p}_3^{\text{c.m.}}| = \frac{\lambda^{1/2}(W^2, m_{a_1}^2, m_p^2)}{2W}, \quad (12)$$

where $\lambda(x, y, z)$ is the Kahlen or triangle function. We take $m_p = 938.27$ MeV and $m_{\pi^-} = 139.57$ MeV in this calculation.

In the effective Lagrangian approach, the sum over polarizations and the Dirac spinors can be easily done thanks to

$$\sum_{s_3} \epsilon^\mu(p_3, s_3) \epsilon^{\nu*}(p_3, s_3) = -g^{\mu\nu} + \frac{p_3^\mu p_3^\nu}{m_{a_1}^2}, \quad (13)$$

$$\sum_{s_4} \bar{u}(p_4, s_4) u(p_4, s_4) = \frac{\not{p}_4 + m_p}{2m_p}. \quad (14)$$

With the formalism and ingredients given above, the calculations of the differential and total cross sections for $\pi^- p \rightarrow \rho^0 \pi^- p$ are straightforward:

$$d\sigma(\pi^- p \rightarrow \rho^0 \pi^- p) = \frac{m_p^2}{256\pi^5 \sqrt{s}} \frac{|\vec{p}_5^*| |\vec{p}_3|}{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \times \left(\frac{1}{2} \sum_{s_2} \sum_{s_3, s_5} |\mathcal{M}(\pi^- p \rightarrow \rho^0 \pi^- p)|^2 \right) dM_{\rho\pi} d\Omega_5^* d\Omega_3, \quad (15)$$

where \vec{p}_5^* and Ω_5^* are the three-momentum and solid angle of the outgoing ρ^0 in the center-of-mass (c.m.) frame of the final $\pi^- \rho^0$ system, while \vec{p}_3 and Ω_3 are the three-momentum and solid angle of the final proton in the c.m. frame of the initial $\pi^- p$ system. In the above equation $M_{\rho\pi}$ is the invariant mass of the final $\pi^- \rho^0$ two-body system, and $s = (p_1 + p_2)^2$ is the invariant mass square of the $\pi^- p$ system.

III. NUMERICAL RESULTS AND DISCUSSION

With the formalism and ingredients given above, the total cross section versus the beam momentum (p_{lab}^1) of the π^- meson for the $\pi^- p \rightarrow a_1^-(1260)p$ reaction is evaluated. The numerical results are shown in Fig. 3 for beam energies p_{lab} from just above the production threshold 2.0 to 5.0 GeV together with the experimental data [27, 28] for comparison. In Fig. 3, the dashed, solid, and dotted curves represent the theoretical results obtained with $\Lambda_\rho = 1.4, 1.5$, and 1.6 GeV, respectively. One can see that the experimental data can be reproduced with a reasonable value of the cutoff parameter $\Lambda_\rho = 1.5 \pm 0.1$ GeV. The experimental data from Ref. [27] were measured at $p_{\text{lab}} = 3.2$ and 4.2 GeV which can be well reproduced with $\Lambda_\rho = 1.5$ GeV. However the experimental data from Ref. [28] at $p_{\text{lab}} = 3.89$ GeV is a few hundred μb larger than the expected value. More experimental measurements are needed to complement the limited data in Refs. [27, 28], and give valuable information about the mechanism of this reaction.

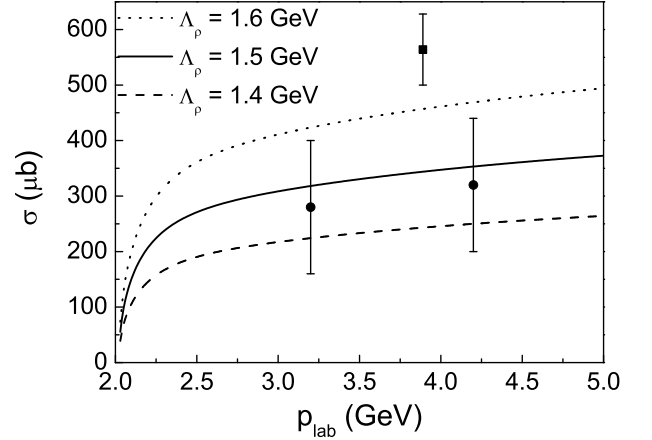


FIG. 3: Total cross section of $\pi^- p \rightarrow a_1^-(1260)p$ reaction versus the incoming π^- beam momentum in the laboratory frame. The circle data points represent the experimental data from Ref. [27], while the square point represents the experimental data from Ref. [28].

Based on the results of the process $\pi^- p \rightarrow a_1^-(1260)p$, we investigate the reaction $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \rho^0 \pi^- p$. The theoretical calculations of the total cross sections of this reaction are shown in Fig. 4, where we take $\Lambda_{a_1} = \Lambda_\rho = 1.5$ GeV for simplicity. It is worth to mention that the numerical results are not sensitive to the value of Λ_{a_1} . In Fig. 4, the dashed, solid, and dotted curves are obtained with $\Gamma_{a_1} = 250, 425$, and 600 MeV, respectively.

In addition to the total cross sections of $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \rho^0 \pi^- p$ reaction, we calculate also the differ-

¹ The relation between W (or s for the case of $\pi^- p \rightarrow \pi^- \rho^0 p$ reaction) and p_{lab} is: $s = W^2 = m_{\pi^-}^2 + m_p^2 + 2m_p \sqrt{m_{\pi^-}^2 + p_{\text{lab}}^2}$.

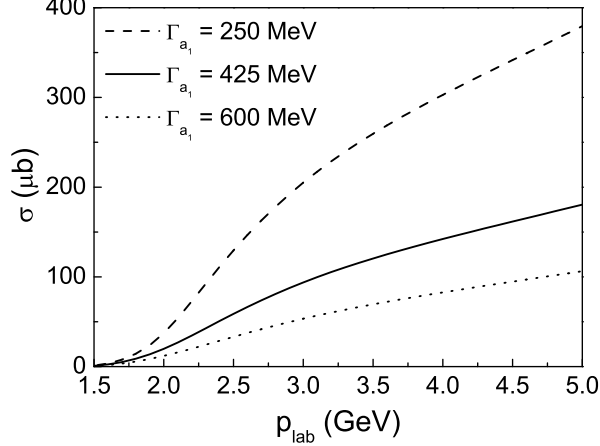


FIG. 4: Total cross section of $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \rho^0 \pi^- p$ reaction versus the π^- beam momentum in the laboratory frame.

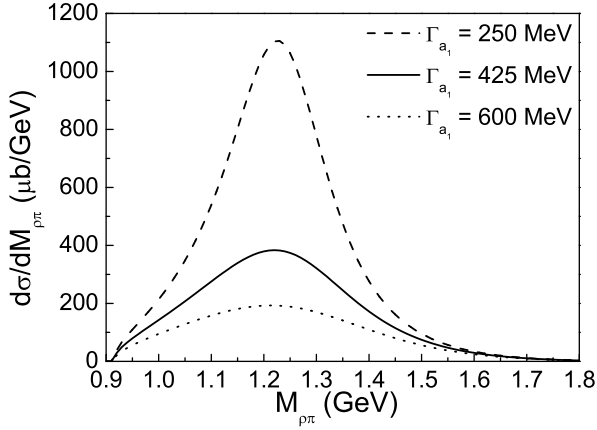


FIG. 5: Invariant mass distributions $d\sigma/dM_{\rho\pi}$ of $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \rho^0 \pi^- p$ reaction at $p_{\text{lab}} = 4$ GeV.

ential cross section for this reaction as a function of $M_{\rho\pi}$ at $p_{\text{lab}} = 4$ GeV. The theoretical results are shown in Fig. 5, where the dashed, solid, and dotted curves are obtained with $\Gamma_{a_1} = 250, 425$, and 600 MeV, respectively. The numerical

results shown in Figs. 4 and 5 could be tested by the future experiments.

IV. SUMMARY

In this work, we have investigated the $\pi^- p \rightarrow a_1^-(1260)p$ and $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \pi^- \rho^0 p$ reactions near threshold within an effective Lagrangian approach. The t -channel ρ^0 meson exchange process is considered with the assumption that the $a_1(1260)$ resonance was dynamically generated from the coupled $K^* \bar{K}$ and $\rho\pi$ channels, from where we can get the coupling of $a_1(1260)$ to $\rho\pi$ channel. The total cross section of $\pi^- p \rightarrow a_1^-(1260)p$ is calculated with the coupling constant of the $a_1(1260)$ to $\rho\pi$ channel obtained from the chiral unitary theory and a reasonable value of cut off parameter Λ_ρ . It is found that the experimental measurement for the $\pi^- p \rightarrow a_1^-(1260)p$ reaction can be fairly reproduced.

Furthermore, the total and differential cross sections of $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \pi^- \rho^0 p$ reaction are also predicted based on the results of the study of the $\pi^- p \rightarrow a_1^-(1260)p$. Because the width of $a_1(1260)$ resonance has large uncertainty, we take different values of Γ_{a_1} for comparison. It is expected that our model calculations can be tested by future experiments.

Finally, we would like to stress that, thanks to the important role played by the t channel ρ^0 exchange in the $\pi^- p \rightarrow a_1^-(1260)p$ reaction, one can reproduce the available experimental data with a reasonable value of the cut off parameter in the form factors. The $\pi^- p \rightarrow a_1^-(1260)p$ and $\pi^- p \rightarrow a_1^-(1260)p \rightarrow \pi^- \rho^0 p$ reactions are important for the study of the $a_1(1260)$ resonance. More and accurate data for these reactions will provide valuable information on the reaction mechanisms and can be used to test our model calculations which should be tied to the nature of the $a_1(1260)$ state. This work provides a vision in this direction.

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- [1] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014). doi:10.1088/1674-1137/38/9/090001
 - [2] E. Klempt and A. Zaitsev, Phys. Rept. **454**, 1 (2007) doi:10.1016/j.physrep.2007.07.006 [arXiv:0708.4016 [hep-ph]].
 - [3] N. Brambilla *et al.*, Eur. Phys. J. C **74**, no. 10, 2981 (2014) doi:10.1140/epjc/s10052-014-2981-5 [arXiv:1404.3723 [hep-ph]].
 - [4] S. K. Choi *et al.* [Belle Collaboration], “Observation of a nar-

- row charmonium - like state in exclusive $B^+ \rightarrow K^+ \pi^+ \pi^- J/\psi$ decays,” Phys. Rev. Lett. **91**, 262001 (2003) [hep-ex/0309032].
- [5] D. Acosta *et al.* [CDF Collaboration], “Observation of the narrow state $X(3872) \rightarrow J/\psi \pi^+ \pi^-$ in $\bar{p}p$ collisions at $\sqrt{s} = 1.96$ TeV,” Phys. Rev. Lett. **93**, 072001 (2004) [hep-ex/0312021].
- [6] V. M. Abazov *et al.* [D0 Collaboration], “Observation and properties of the $X(3872)$ decaying to $J/\psi \pi^+ \pi^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV,” Phys. Rev. Lett. **93**, 162002 (2004) [hep-ex/0405004].
- [7] M. Ablikim *et al.* [BESIII Collaboration], “Observation of

- a Charged Charmoniumlike Structure in $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ at $\sqrt{s}=4.26$ GeV,” Phys. Rev. Lett. **110**, 252001 (2013) [arXiv:1303.5949 [hep-ex]].
- [8] Z. Q. Liu *et al.* [Belle Collaboration], “Study of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ and Observation of a Charged Charmoniumlike State at Belle,” Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].
- [9] C. Adolph *et al.* [COMPASS Collaboration], Phys. Rev. Lett. **115**, no. 8, 082001 (2015) doi:10.1103/PhysRevLett.115.082001 [arXiv:1501.05732 [hep-ex]].
- [10] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **117**, no. 2, 022003 (2016) doi:10.1103/PhysRevLett.117.022003 [arXiv:1602.07588 [hep-ex]].
- [11] M. Alekseev *et al.* [COMPASS Collaboration], Phys. Rev. Lett. **104**, 241803 (2010) doi:10.1103/PhysRevLett.104.241803 [arXiv:0910.5842 [hep-ex]].
- [12] L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005) doi:10.1103/PhysRevD.72.014002 [hep-ph/0503273].
- [13] L. Roca, A. Hosaka and E. Oset, Phys. Lett. B **658**, 17 (2007) doi:10.1016/j.physletb.2007.10.035 [hep-ph/0611075].
- [14] H. Nagahiro, L. Roca, A. Hosaka and E. Oset, Phys. Rev. D **79**, 014015 (2009) doi:10.1103/PhysRevD.79.014015 [arXiv:0809.0943 [hep-ph]].
- [15] M. Mikhasenko, B. Ketzer and A. Sarantsev, Phys. Rev. D **91**, no. 9, 094015 (2015) doi:10.1103/PhysRevD.91.094015 [arXiv:1501.07023 [hep-ph]].
- [16] F. Aceti, L. R. Dai and E. Oset, arXiv:1606.06893 [hep-ph].
- [17] W. Wang and Z. X. Zhao, Eur. Phys. J. C **76**, no. 2, 59 (2016) doi:10.1140/epjc/s10052-016-3900-8 [arXiv:1511.06998 [hep-ph]].
- [18] R. Machleidt, K. Holinde and C. Elster, Phys. Rept. **149**, 1 (1987). doi:10.1016/S0370-1573(87)80002-9
- [19] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
- [20] M. Doring, C. Hanhart, F. Huang, S. Krewald, U.-G. Meissner and D. Ronchen, Nucl. Phys. A **851**, 58 (2011) doi:10.1016/j.nuclphysa.2010.12.010 [arXiv:1009.3781 [nucl-th]].
- [21] D. Ronchen *et al.*, Eur. Phys. J. A **49**, 44 (2013) doi:10.1140/epja/i2013-13044-5 [arXiv:1211.6998 [nucl-th]].
- [22] K. Tsushima, A. Sibirtsev and A. W. Thomas, Phys. Rev. C **62**, 064904 (2000) doi:10.1103/PhysRevC.62.064904 [nucl-th/0004011].
- [23] A. M. Gasparyan, J. Haidenbauer, C. Hanhart and J. Speth, Phys. Rev. C **68**, 045207 (2003) doi:10.1103/PhysRevC.68.045207 [nucl-th/0307072].
- [24] J. J. Xie and B. S. Zou, Phys. Lett. B **649**, 405 (2007) doi:10.1016/j.physletb.2007.04.035 [nucl-th/0701021].
- [25] J. J. Xie, B. S. Zou and H. C. Chiang, Phys. Rev. C **77**, 015206 (2008) doi:10.1103/PhysRevC.77.015206 [arXiv:0705.3950 [nucl-th]].
- [26] J. J. Xie, Phys. Rev. C **92**, no. 6, 065203 (2015) doi:10.1103/PhysRevC.92.065203 [arXiv:1509.06196 [nucl-th]].
- [27] S. U. Chung, O. I. Dahl, J. Kirz and D. H. Miller, Phys. Rev. **165**, 1491 (1968). doi:10.1103/PhysRev.165.1491
- [28] P. G. Wohlmuth, K. Abe, A. D. Johnson and V. J. Stenger, Nucl. Phys. B **18**, 505 (1970). doi:10.1016/0550-3213(70)90132-X